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Tachyon condensation on the intersecting brane-antibrane system

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Abstract: We generalize our study of the tachyon condensation on the brane-antibrane system ([1]) to the intersecting brane-antibrane system. The supergravity solutions of the intersecting brane-antibrane system are characterized by five parameters. We relate these parameters to the microscopic physical parameters, namely, the number of Dp-branes (N_1), the number of $\bar{\mathrm{D}}p$-branes ($\bar{N}_1$), the number of D$(p-4)$-branes (N_2), the number of $\bar{\mathrm{D}}(p-4)$-branes ($\bar{N}_2$) and the tachyon vev T. We show that the solution and the ADM mass capture all the required properties and give a correct description of the tachyon condensation for the intersecting brane-antibrane system.

Keywords: D-branes, Tachyon Condensation.

A coincident D-brane–antiD-brane pair (or a non-BPS D-brane) in Type II string theories is known to be unstable which is characterized, for example, by the presence of tachyonic mode on the D-brane world-volume [2]. As a result, these systems decay and the decay occurs by a process known as tachyon condensation [3]. Tachyon condensation is well understood in the open string description using either the string field theory approach [4, 5] or the tachyon effective action approach [6] on the brane. In [1] we have obtained a closed string (or supergravity) understanding of this process. We have interpreted [7] the previously known [8, 9] non-supersymmetric, three parameter supergravity solutions with a symmetry $\mathrm{ISO}(p,1)\times\mathrm{SO}(9-p)$ in ten space-time dimensions as the coincident Dp–$\bar{\mathrm{D}}p$ system. Then the three parameters in this solution were related to the microscopic physical parameters, namely, the number of Dp-branes (N), the number of $\bar{\mathrm{D}}p$-branes ($\bar{N}$) and the tachyon vev of the Dp–$\bar{\mathrm{D}}p$ system. Using these relations we have calculated the ADM mass and have shown that the solution and the ADM mass capture all the required properties and give a correct description of the tachyon condensation advocated by Sen [3] on the D-$\bar{\mathrm{D}}$ system.

In this paper we generalize our previous study [1] of the tachyon condensation on the brane-antibrane system to the intersecting brane-antibrane system. The supergravity solution in this case is again non-supersymmetric and has the isometry $\mathrm{ISO}(p-4,1)\times\mathrm{SO}(4)\times\mathrm{SO}(9-p)$ which can obviously be identified as the intersecting brane-antibrane system or more precisely the intersecting Dp-$\bar{\mathrm{D}}p$ and D$(p-4)$-$\bar{\mathrm{D}}(p-4)$ system for $3\leq p\leq 6$. We find that the solution is characterized by five parameters and once the solution is realized to represent intersecting Dp-$\bar{\mathrm{D}}p$ and D$(p-4)$-$\bar{\mathrm{D}}(p-4)$ system, we can immediately infer that the parameters must be related to the microscopic physical parameters N_1, the number of Dp-branes, $\bar{N}_1$, the number of $\bar{\mathrm{D}}p$-branes, N_2, the number of D$(p-4)$-branes, $\bar{N}_2$, the number of $\bar{\mathrm{D}}(p-4)$-branes and the tachyon vev T.[1] The more exact relationships between the supergravity parameters and the microscopic physical parameters can be obtained by examining how the solution reduces to the supersymmetric configuration which corresponds to the four cases as follows. (a) The intersecting N_1 Dp-branes and N_2 D$(p-4)$-branes (when both $\bar{N}_1$ and $\bar{N}_2$ are zero). If in addition to $\bar{N}_1=\bar{N}_2=0$ we also have $N_2=0$ (or $N_1=0$), we still get a BPS configuration of half susy instead of quarter susy (as is the case for the intersecting Dp/D$(p-4)$ brane configuration) which is N_1 Dp-branes (or delocalized N_2 D$(p-4)$-branes). When all the N's are zero, which is a trivial case, we get the maximally supersymmetric flat space-time. (b) The intersecting $\bar{N}_1$ $\bar{\mathrm{D}}p$-branes and $\bar{N}_2$ $\bar{\mathrm{D}}(p-4)$-branes (when both N_1 and N_2 are zero). Similar to the previous case here also if in addition to $N_1=N_2=0$, we have $\bar{N}_2=0$ (or $\bar{N}_1=0$) we get half susy configuration which is $\bar{N}_1$ $\bar{\mathrm{D}}p$-branes (or delocalized $\bar{N}_2$ $\bar{\mathrm{D}}(p-4)$-branes). (c) The intersecting N_1 Dp-branes and $\bar{N}_2$ $\bar{\mathrm{D}}(p-4)$-branes (when both $\bar{N}_1$ and N_2 are zero). This case actually preserves also one quarter of spacetime supersymmetry which seems impossible from a first look.[2] Again similar to the previous case here also if in

[1] So long as the bulk configuration is concerned, the worldvolume fields (in particular the tachyon) don't need to satisfy their respective worldvolume equations of motion (for example, we can put worldvolume scalars and tachyon to constants and other fields to zero) in the spirit, for example, of [7]. In other words, they can be off-shell. In this way, the tachyon vev will appear as a parameter labelling the solution.

[2] The (p,p') system of D-branes with $p-p'=4$ is very special and when one calculates the one-loop interaction amplitude between them (for example, see [10]), the R-R amplitude has zero contribution while

addition to $\bar{N}_1 = N_2 = 0$, we have $\bar{N}_2 = 0$ (or $N_1 = 0$) we get half susy configuration which is N_1 Dp-branes (or delocalized $\bar{N}_2$ $\bar{\mathrm{D}}(p-4)$-branes). (d) This is similar in sprit to the case (c) but now with $\bar{N}_1$ and N_2 non-zero. Other special cases can be discussed accordingly.

When none of the N's are zero, in general, the solution is not supersymmetric and there is a tachyon on the world-volume of the intersecting branes.[3] However we can always get BPS configuration at the end of tachyon condensation given the above discussion and the one in footnote 5. When $N_1 = \bar{N}_1$ or $N_2 = \bar{N}_2$, we will restore one half of susy at the end of tachyon condensation. When the two hold simultaneously, we end up with a flat spacetime at the end of tachyon condensation with maximal supersymmetry. For all the other cases, one quarter of susy is preserved at the end of tachyon condensation. For example, when $N_1 > \bar{N}_1$ and $N_2 > \bar{N}_2$ we expect to have intersecting $(N_1 - \bar{N}_1)$ Dp-branes with $(N_2 - \bar{N}_2)$ D$(p-4)$-branes at the end of tachyon condensation which is supersymmetric (quarter BPS), on the other hand, when $N_1 = \bar{N}_1$ we expect to have delocalized $(N_2 - \bar{N}_2)$ D$(p-4)$-branes at the end of tachyon condensation which is half BPS and so on. The recognition for having a supersymmetric background at the end of tachyon condensation is crucial and we use all these information to relate the supergravity parameters to the microscopic physical parameters.

Now in order to understand the tachyon condensation, we look at the expression of the total ADM mass of the intersecting brane-antibrane supergravity solution representing the total energy of the system. We then express this total energy in terms of the five microscopic physical parameters namely, N_1, $\bar{N}_1$, N_2, $\bar{N}_2$ and T of the intersecting Dp-$\bar{\mathrm{D}}p$ and D$(p-4)$-$\bar{\mathrm{D}}(p-4)$ system using the aforementioned relations. The total energy can be seen to be equal to or less than the sum of the masses of N_1 Dp-branes, $\bar{N}_1$ $\bar{\mathrm{D}}p$-branes, N_2 D$(p-4)$-branes and $\bar{N}_2$ $\bar{\mathrm{D}}(p-4)$-branes indicating the presence of tachyon contributing the negative potential energy to the system. We will see that the energy expression gives the right picture of tachyon condensation as is expected of an intersecting brane-antibrane system. We will reproduce all the expected results from this general mass formula under

the NS-NS amplitude has two terms cancelled for this case. Since the NS-NS amplitude is insensitive to the sign of the charges carried by the individual D-brane, therefore the force acting between the two branes vanishes, independent of the sign of their charges. Such a no-force condition indicates that there is a certain fraction of spacetime supersymmetry preserved. Further when two kinds of D-branes with different dimensionality intersect, the spacetime supersymmetry preserved is determined by the following two equations: $\epsilon_1 = \eta\Gamma^0\Gamma^1\cdots\Gamma^p\,\epsilon_2$ and $\epsilon_1 = \eta'\Gamma^0\Gamma^1\cdots\Gamma^{p'}\,\epsilon_2$ with Γ^μ the ten dimensional γ-matrix, and ϵ_1 and ϵ_2 the two Majorana-Weyl supersymmetry parameters in IIA/IIB string theory (In IIA, p and p' are both even while in IIB they are odd). Here $\eta = \pm$ and $\eta' = \pm$ label the sign of the charge carried by the corresponding branes, respectively. Assuming $p > p'$ and defining $\Gamma = \Gamma^{p'+1}\cdots\Gamma^p$, one can show that only for $\Gamma^2 = I$ with I the unit matrix, i.e., for, $(-1)^{(p-p')(p-p'+1)/2} = 1$, a quarter of susy is preserved, independent of the values of η and η'. In a given theory, $p - p'$ is even and the above condition implies $p - p' = 4, 8$. As $p = p'$, Γ is a unit matrix, then only for $\eta = \eta'$, one half of susy can be preserved. Here we discuss the so-called threshold bound states. For nonthreshold bound state, one half of susy can be preserved for $p - p' = 2$ based on U-duality.

[3]The tachyon can be expressed as $T = \begin{pmatrix} T_1 & T_2 \\ T_3 & T_4 \end{pmatrix}$. Here T_1 is in the bi-fundamental $(N_1, \bar{N}_1)$ of gauge group $\mathrm{U}(N_1) \times \mathrm{U}(\bar{N}_1)$, T_2 in $(N_1, \bar{N}_2)$ of gauge group $\mathrm{U}(N_1) \times \mathrm{U}(\bar{N}_2)$, T_3 in $(N_2, \bar{N}_1)$ of gauge group $\mathrm{U}(N_2) \times \mathrm{U}(\bar{N}_1)$ and T_4 in $(N_2, \bar{N}_2)$ of gauge group $\mathrm{U}(N_2) \times \mathrm{U}(\bar{N}_2)$. The tachyon used in the discussion is actually $(Tr\,(T\bar{T}))^{1/2}$ with a proper normalization.

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various special limits at the top and at the bottom of the tachyon potential. We will also show how the various known BPS supergravity configurations can be reproduced in these special limits.

The non-supersymmetric intersecting Dp/D$(p-4)$ supergravity solution having isometry ISO$(p-4,1)\times$SO$(4)\times$SO$(9-p)$ which can also be identified as the intersecting Dp-$\bar{\rm D}p$ and D$(p-4)$-$\bar{\rm D}(p-4)$ has the form in space-time dimension $d=10$ as,[4]

$$\begin{aligned}
ds^2 &= F_1^{-\frac{7-p}{8}} F_2^{-\frac{11-p}{8}} \left(-dt^2 + \sum_{i=1}^{p-4} dx_i^2\right) + F_1^{-\frac{7-p}{8}} F_2^{\frac{p-3}{8}} \sum_{j=p-3}^{p} dx_j^2 + \\
&\quad + \left(H\tilde{H}\right)^{\frac{2}{7-p}} F_1^{\frac{p+1}{8}} F_2^{\frac{p-3}{8}} \left(dr^2 + r^2 d\Omega_{8-p}^2\right) \\
e^{2\phi} &= F_1^{\frac{3-p}{2}} F_2^{\frac{7-p}{2}} \left(\frac{H}{\tilde{H}}\right)^{2\delta} \\
F_{[8-p]} &= b\,\mathrm{Vol}(\Omega_{8-p}) \\
F_{[12-p]} &= c\,\mathrm{Vol}(\Omega_{8-p}) \wedge dx_{p-3} \wedge \cdots \wedge dx_p\,.
\end{aligned} \tag{1}$$

Note that in the above we have written the metric in the Einstein frame. The Vol(Ω_{8-p}) represents the volume form of the unit $(8-p)$-dimensional sphere. Also the various functions appeared in the solution are defined below,

$$\begin{aligned}
F_1 &= \cosh^2\theta_1 \left(\frac{H}{\tilde{H}}\right)^{\alpha_1} - \sinh^2\theta_1 \left(\frac{\tilde{H}}{H}\right)^{\beta_1} \\
F_2 &= \cosh^2\theta_2 \left(\frac{H}{\tilde{H}}\right)^{\alpha_2} - \sinh^2\theta_2 \left(\frac{\tilde{H}}{H}\right)^{\beta_2} \\
H &= 1 + \frac{\omega^{7-p}}{r^{7-p}}\,, \qquad \tilde{H} = 1 - \frac{\omega^{7-p}}{r^{7-p}}\,.
\end{aligned} \tag{2}$$

Here α_1, β_1, α_2, β_2, δ and ω are integration constants. b and c are the charge parameters related to the net charges of the Dp-$\bar{\rm D}p$ and D$(p-4)$-$\bar{\rm D}(p-4)$-brane systems respectively. However, not all the parameters are independent and they are related as,

$$\begin{aligned}
\alpha_1 - \beta_1 &= \frac{p-3}{2}\delta\,, \qquad b = (7-p)(\alpha_1+\beta_1)\omega^{7-p}\sinh 2\theta_1 \\
\alpha_2 - \beta_2 &= \frac{p-7}{2}\delta\,, \qquad c = (7-p)(\alpha_2+\beta_2)\omega^{7-p}\sinh 2\theta_2
\end{aligned} \tag{3}$$

and from the consistency of equations of motion we also get

$$(\alpha_1+\beta_1)^2 + (\alpha_2+\beta_2)^2 + \left(4 - \frac{(p-3)^2}{4} - \frac{(p-7)^2}{4}\right)\delta^2 = 8\frac{8-p}{7-p}\,. \tag{4}$$

So, treating $(\alpha_1+\beta_1)\equiv\Delta_1$ and $(\alpha_2+\beta_2)\equiv\Delta_2$ as independent, there are actually five independent parameters in the solution, namely, θ_1, θ_2, ω, Δ_1 and Δ_2. We will relate each of these parameters to the physical microscopic parameters of the intersecting Dp-$\bar{\rm D}p$ and

[4]This configuration has also been considered previously in [11, 12] but in a rather different notations.

D$(p-4)$-$\bar{\text{D}}(p-4)$ system. But before we do that we would like to make some comments. First of all, we mention that as $r \to \infty$, both H and $\tilde{H} \to 1$, and so, F_1, $F_2 \to 1$. The solution is therefore asymptotically flat. Also, note that the solution (1) represents magnetically charged intersecting brane-antibrane system. To obtain the electrically charged ones we just make a transformation $F_{[8-p]} \to e^{\frac{3-p}{2}\phi} * F_{[8-p]}$ and $F_{[12-p]} \to e^{\frac{7-p}{2}\phi} * F_{[12-p]}$, where $*$ represents the Hodge dual. Using these the gauge fields for the electrically charged solution can be written as,

$$\begin{aligned} A_{[p+1]} &= \sinh\theta_1 \cosh\theta_1 \left(\frac{C_1}{F_1}\right) dt \wedge dx_1 \wedge \cdots \wedge dx_p \\ A_{[p-3]} &= \sinh\theta_2 \cosh\theta_2 \left(\frac{C_2}{F_2}\right) dt \wedge dx_1 \wedge \cdots \wedge dx_{p-4}\,, \end{aligned} \tag{5}$$

where C_1 and C_2 are defined as,

$$\begin{aligned} C_1 &= \left(\frac{H}{\tilde{H}}\right)^{\alpha_1} - \left(\frac{\tilde{H}}{H}\right)^{\beta_1} \\ C_2 &= \left(\frac{H}{\tilde{H}}\right)^{\alpha_2} - \left(\frac{\tilde{H}}{H}\right)^{\beta_2}\,. \end{aligned} \tag{6}$$

We note from (2) that the solution has a potential singularity at $r = \omega$ and we will work only in the physically relevant region $r > \omega$. Also, without any loss of generality we will choose all of the parameters Δ_1, Δ_2 to be ≥ 0. We remark that the solution is non-supersymmetric can be seen from the $(H\tilde{H})^{2/(7-p)}$ factor in the last term of the metric in eq. (1). This is consistent with our interpretation of the solution to be intersecting brane-antibrane system. We can now express the parameter b and c in terms of the number of branes-antibranes using eq. (1) as follows,

$$\begin{aligned} Q_0^p\,(N_1 - \bar{N}_1) &= \frac{b\Omega_{8-p}}{\sqrt{2}\kappa_0} \quad \Rightarrow \quad b = \frac{\sqrt{2}\kappa_0 Q_0^p (N_1 - \bar{N}_1)}{\Omega_{8-p}} \\ Q_0^{p-4}\,(N_2 - \bar{N}_2) &= \frac{c\Omega_{8-p} V_4}{\sqrt{2}\kappa_0} \quad \Rightarrow \quad c = \frac{\sqrt{2}\kappa_0 Q_0^{p-4} (N_2 - \bar{N}_2)}{\Omega_{8-p} V_4}\,, \end{aligned} \tag{7}$$

where $Q_0^p = (2\pi)^{(7-2p)/2} \alpha'^{(3-p)/2}$ is the unit charge on the Dp-brane and similarly Q_0^{p-4} is the unit charge on the D$(p-4)$-brane. $\sqrt{2}\kappa_0 = (2\pi)^{7/2}\alpha'^2$ is related to 10 dimensional Newton's constant. V_4 is the compact volume of the four directions x_{p-3} to x_p. $\Omega_n = 2\pi^{(n+1)/2}/\Gamma((n+1)/2)$. Note that $b \to 0$ as $N_1 \to \bar{N}_1$ and $c \to 0$ as $N_2 \to \bar{N}_2$ as expected.

For the solution (1), the supersymmetry will be restored if and only if $H\tilde{H} \to 1$ which always requires $\omega^{7-p} \to 0$. As we have already stated in the beginning we have the following cases for which supersymmetry will be restored: (1) $N_1 = N_2 = 0$, or, $\bar{N}_1 = \bar{N}_2 = 0$, or, $N_1 = \bar{N}_2 = 0$, or, $\bar{N}_1 = N_2 = 0$ (or, $N_1 = N_2 = \bar{N}_1 = \bar{N}_2 = 0$ which is the trivial case). (2) When none of the groups are zero, we can still have supersymmetric configuration at the end of tachyon condensation for which the number of susy restored depends on the initial values of $N_1, N_2, \bar{N}_1$ and $\bar{N}_2$ as discussed earlier. For example, when $N_1 = \bar{N}_1$ and

$N_2 = \bar{N}_2$, we expect to get an empty space-time at the end of tachyon condensation with maximal supersymmetry. What is important is the recognition that for all these cases, $\omega^{7-p} \to 0$. This observation will be crucial to relate ω as well as other parameters in terms of the physical microscopic parameters of the brane-antibrane system. We also point out that at the top of the tachyon potential when ω^{7-p} does not go to zero, it must give the correct number of branes so that the mass formula can be correctly reproduced. All these information can be captured in a single formula for ω^{7-p} as follows,

$$(7-p)\omega^{7-p} = \sqrt{\frac{7-p}{2(8-p)}} \frac{2\kappa_0^2}{\Omega_{8-p}} T_p \left[\sqrt{N_1\bar{N}_1} + a\sqrt{N_2\bar{N}_2}\right] \cos T\,. \tag{8}$$

In the above $T_p = (2\pi)^{-p}\alpha'^{-(p+1)/2}$ is the tension of a Dp-brane. The factor $2\kappa_0^2 T_p/\Omega_{8-p}$ in front is kept so that it will give the correct mass of the system. Also we have defined $a = T_{p-4}/(V_4 T_p) = (2\pi\sqrt{\alpha'})^4/V_4$ relating the tensions of a Dp-brane and a D$(p-4)$-brane. We also point out that T in the above represents the tachyon vev and we have taken $T = 0$ as the top of the tachyon potential and $T = \pi/2$ as the bottom of the potential. Thus all the cases we have discussed above is contained in ω^{7-p} in eq. (8).

Now having obtained the form of ω^{7-p} in terms of the microscopic physical parameters, we would like to obtain similar relations for the other parameters as well. But first we will try to relate δ, Δ_1 and Δ_2 in terms of the microscopic parameters. But before that we observe that the total ADM mass of the intersecting Dp-$\bar{\mathrm{D}}p$ and D$(p-4)$-$\bar{\mathrm{D}}(p-4)$ can be obtained from the supergravity configuration (1) using the formula given in [13] as,

$$M = \frac{\Omega_{8-p}}{2\kappa_0^2}(7-p)\omega^{7-p}\left[\left(\Delta_1 \cosh 2\theta_1 + \frac{p-3}{2}\delta\right) + \left(\Delta_2 \cosh 2\theta_2 + \frac{p-7}{2}\delta\right)\right]. \tag{9}$$

This clearly shows that the ADM mass or the total energy of the system breaks up into two parts corresponding to the Dp-$\bar{\mathrm{D}}p$ and the D$(p-4)$-$\bar{\mathrm{D}}(p-4)$-brane systems. From our earlier experience on the tachyon condensation on the brane-antibrane system we find that in this case the parameters Δ_1 and Δ_2 satisfy the following relations in terms of δ

$$\begin{aligned}
\Delta_1^2 &= \frac{(p-3)^2}{4}\delta^2 - \frac{(p-3)\delta}{\gamma A}\sqrt{\frac{(N_1-\bar{N}_1)^2}{\cos^2 T} + 4N_1\bar{N}_1\cos^2 T} + \frac{4N_1\bar{N}_1\cos^2 T}{\gamma^2 A^2}\\
\Delta_2^2 &= \frac{(p-7)^2}{4}\delta^2 - \frac{(p-7)a\delta}{\gamma A}\sqrt{\frac{(N_2-\bar{N}_2)^2}{\cos^2 T} + 4N_2\bar{N}_2\cos^2 T} + \frac{4a^2N_2\bar{N}_2\cos^2 T}{\gamma^2 A^2}\,,
\end{aligned} \tag{10}$$

where $\gamma = \sqrt{(7-p)/2(8-p)}$, $\Delta_1 = \alpha_1 + \beta_1$, $\Delta_2 = \alpha_2 + \beta_2$ and $A = \sqrt{N_1\bar{N}_1} + a\sqrt{N_2\bar{N}_2}$. Then using the parameter relation (4) we find that the parameter δ satisfy the following quadratic relation where only the $\delta < 0$ root will be relevant for our discussion

$$\begin{aligned}
\delta^2 - \frac{\delta}{4\gamma A}\Bigg[(p-3)&\sqrt{\frac{(N_1-\bar{N}_1)^2}{\cos^2 T} + 4N_1\bar{N}_1\cos^2 T} +\\
&+ a(p-7)\sqrt{\frac{(N_2-\bar{N}_2)^2}{\cos^2 T} + 4N_2\bar{N}_2\cos^2 T}\Bigg] -\\
&- \frac{(N_1\bar{N}_1 + a^2N_2\bar{N}_2)\sin^2 T + 2a\sqrt{N_1N_2\bar{N}_1\bar{N}_2}}{\gamma^2 A^2} = 0\,.
\end{aligned} \tag{11}$$

Thus δ is given entirely in terms of the microscopic physical parameters. As $T \to 0$, the δ approaches a finite negative value while as $T \to \pi/2$, $\delta = -4(A/\gamma)\cos T/[(p-3)|N_1 - \bar{N}_1| + a(p-7)|N_2 - \bar{N}_2|]$ if not all the four $N_1, N_2, \bar{N}_1, \bar{N}_2$ are equal and if so, then $\delta = -1/\gamma$. Once δ is known we can determine Δ_1 and Δ_2 in terms of the microscopic parameters as well. After we determine the forms of Δ_1 and Δ_2 then using the form of ω^{7-p} given in (8) we can easily relate the parameters θ_1 and θ_2 to the microscopic physical parameters using (3) and (7) as,

$$\begin{aligned} \sinh 2\theta_1 &= \frac{|N_1 - \bar{N}_1|}{\gamma \Delta_1 A \cos T}, \\ \sinh 2\theta_2 &= \frac{a|N_2 - \bar{N}_2|}{\gamma \Delta_2 A \cos T}, \end{aligned} \tag{12}$$

where we have assumed both θ_1 and θ_2 to be ≥ 0. We note from (12) that as $N_1 \to \bar{N}_1$, $\theta_1 \to 0$. We have also seen it before from eq. (7) that $N_1 \to \bar{N}_1$ implies $b \to 0$ and so, $\theta_1 \to 0$ implies the charge $b \to 0$. Similarly, $\theta_2 \to 0$ implies the charge $c \to 0$. Note from eq. (4) that the values of $|\Delta_1|, |\Delta_2|$ and $|\delta|$ are all bounded from the above, therefore from (12) we see that both θ_1 and θ_2 blow up at the end of the tachyon condensation.

Now all the quantities in the ADM mass expression given in eq. (9) are known and so, substituting ω^{7-p}, Δ_1, Δ_2 and δ we find that the mass expression has a very simplified form given by

$$\begin{aligned} M &= T_p \left[\sqrt{(N_1 - \bar{N}_1)^2 + 4N_1\bar{N}_1 \cos^4 T} + a\sqrt{(N_2 - \bar{N}_2)^2 + 4N_2\bar{N}_2 \cos^4 T} \right] \\ &\leq T_p \left[(N_1 + \bar{N}_1) + a(N_2 + \bar{N}_2) \right]. \end{aligned} \tag{13}$$

We thus note that the total mass per unit p-brane volume of the system is less or equal to the sum of the those of N_1 Dp-branes, $\bar{N}_1$ $\bar{\text{D}}p$-branes, N_2 D$(p-4)$-branes, $\bar{N}_2$ $\bar{\text{D}}(p-4)$-branes and the difference is the tachyon potential energy per unit p-brane volume $V(T)$ which is negative. We can easily check that $T = 0$ gives the maximum of the energy, therefore the maximum of the tachyon potential (which is zero) while $T = \pi/2$ gives the corresponding minima. We want to point out that eq. (13) is consistent with our previous experience for a simple brane-antibrane system discussed in [1] and the special feature for the present system under consideration is that no interaction exists between two D-branes with their dimensionality differing by four as discussed in footnote 5.

We will now check one by one whether the above mass formula produces all the required properties of the solution and the tachyon condensation. At $T = 0$, i.e. at the top of the tachyon potential, $\cos T = 1$ and we have from above $M = T_p\left[(N_1 + \bar{N}_1) + a(N_2 + \bar{N}_2)\right]$, producing the expected result. It is also easy to see from (8) that at that point ω^{7-p} does not go to zero indicating that the corresponding solution breaks all the supersymmetry as it should be. Note that in this case it does not matter whether we have (i) $N_1 \geq \bar{N}_1$ and $N_2 \geq \bar{N}_2$, (ii) $N_1 \leq \bar{N}_1$ and $N_2 \leq \bar{N}_2$, (iii) $N_1 \geq \bar{N}_1$ and $N_2 \leq \bar{N}_2$ or (iv) $N_1 \leq \bar{N}_1$ and $N_2 \geq \bar{N}_2$, in all four cases ω^{7-p} has the same value. Also at $T = \pi/2$ i.e. at the bottom of the tachyon potential we get from above $M = T_p(|N_1 - \bar{N}_1| + a|N_2 - \bar{N}_2|)$ again producing the expected result. Here also it does not matter whether we have cases (i) to

(iv) above, we always have ω^{7-p} going to zero, indicating that we have a supersymmetric configurations. In fact, for (i) we have intersecting $(N_1 - \bar{N}_1)$ Dp branes and $(N_2 - \bar{N}_2)$ D$(p-4)$ branes (when none of the inequalities are saturated), $(N_1 - \bar{N}_1)$ Dp-branes (when the second inequality is saturated) and $(N_2 - \bar{N}_2)$ delocalized D$(p-4)$-branes (when the first inequality is saturated). Similarly for case (ii). For case (iii) we have intersecting $(N_1 - \bar{N}_1)$ Dp branes and $(\bar{N}_2 - N_2)$ $\bar{\mathrm{D}}(p-4)$ branes (when none of the inequalities are saturated), $(N_1 - \bar{N}_1)$ Dp-branes (when the second inequality is saturated) and $(\bar{N}_2 - N_2)$ delocalized $\bar{\mathrm{D}}(p-4)$-branes (when the first inequality is saturated). Similarly for case (iv).

Now we will discuss in a bit detail how one can recover the supersymmetric intersecting brane configuration at the end of tachyon condensation. Let us first discuss the case $N_1 > \bar{N}_1$ and $N_2 > \bar{N}_2$. Since for $T \to \pi/2$, $\omega^{7-p} \to 0$, both H and $\tilde{H}$ goes to unity as can be seen from eq. (2). Also we notice from (12) that both θ_1 and θ_2 goes to ∞. So, if we take a limit $(\alpha_1 + \beta_1)\sinh\theta_1 \to \epsilon_1^{-1}$, $(\alpha_2 + \beta_2)\sinh\theta_2 \to \epsilon_2^{-1}$ and $\omega^{7-p} \to \epsilon_1 \bar{\omega}_1^{7-p}$ and $\omega^{7-p} \to \epsilon_2 \bar{\omega}_2^{7-p}$ for some dimensionless parameters ϵ_1, $\epsilon_2 \to 0$, with $\bar{\omega}_1^{7-p}$ and $\bar{\omega}_2^{7-p}$ = finite, then we get from (3) $b = (7-p)\bar{\omega}_1^{7-p}$ and $c = (7-p)\bar{\omega}_2^{7-p}$. Both F_1 and F_2 in eq. (2) would then reduce to some harmonic functions

$$
\begin{aligned}
F_1 &\to \bar{H}_1 = 1 + \frac{\bar{\omega}_1^{7-p}}{r^{7-p}} \\
F_2 &\to \bar{H}_2 = 1 + \frac{\bar{\omega}_2^{7-p}}{r^{7-p}}
\end{aligned} \tag{14}
$$

with $\bar{\omega}_1^{7-p} = b/(7-p)$ and $\bar{\omega}_2^{7-p} = c/(7-p)$. The corresponding configuration as can be seen from (1) with eq. (14) is an intersecting BPS configuration of $(N_1 - \bar{N}_1)$ Dp branes with $(N_2 - \bar{N}_2)$ D$(p-4)$ branes. One quarter of susy is restored. The other three cases $N_1 < \bar{N}_1$ and $N_2 < \bar{N}_2$, or, $N_1 > \bar{N}_1$ and $N_2 < \bar{N}_2$, or, $N_1 < \bar{N}_1$ and $N_2 > \bar{N}_2$ will be very similar and we will not repeat the discussion. However when one of them is equal i.e. for the case $N_1 > \bar{N}_1$ (or $N_1 < \bar{N}_1$) and $N_2 = \bar{N}_2$, we find from eq. (12) that $\theta_2 = 0$ which implies from eq. (2) that $F_2 \to 1$ (because H and $\tilde{H}$ go to unity). So, in this case we will have just one harmonic function $\bar{H}_1$ rather than two. The corresponding solution can be easily checked from (1) to get reduced to $(N_1 - \bar{N}_1)$ Dp-branes (or $(\bar{N}_1 - N_1)$ $\bar{\mathrm{D}}p$-branes). Now one half of susy is restored. Similarly for the other case i.e. when $N_1 = \bar{N}_1$ and $N_2 > \bar{N}_2$ (or $N_2 < \bar{N}_2$). The tachyon condensation can also be seen for the special case of $N_1 = \bar{N}_1$ and $N_2 = \bar{N}_2$. For this case, at the top of the potential ($T = 0$) we get from (13) $M = 2T_p(N_1 + aN_2)$ as expected and the corresponding configuration breaks all the supersymmetry. However, at the end of tachyon condensation i.e. at $T = \pi/2$, $M = 0$ corresponding to an empty space-time preserving all supersymmetry. In fact in this case we see from (8) that ω^{7-p} vanishes and so $\bar{H}_1$, $\bar{H}_2 = 1$. For this case, all susy is restored.

It is also easy to check that the mass formula produces the correct result when $N_1 = \bar{N}_1 = 0$ or $N_2 = \bar{N}_2 = 0$ or both. For the first case, we have from (13) $M = a(N_2 + \bar{N}_2)T_{p-4}/V_4$ at the top of the potential and $M = a|N_2 - \bar{N}_2|T_{p-4}/V_4$ at the bottom. For the second case we have $M = (N_1 + \bar{N}_1)T_p$ at the top and $M = |N_1 - \bar{N}_1|T_p$ at the bottom as expected. When both $N_1 = \bar{N}_1 = 0$ and $N_2 = \bar{N}_2 = 0$ we get $M = 0$ as expected. It is not difficult to check that the space-time configuration at the end of tachyon condensation in these three cases indeed reduce to BPS $(N_2-\bar{N}_2)$ delocalized D$(p-4)$-branes (or $(\bar{N}_2-N_2)$ delocalized $\bar{\mathrm{D}}(p-4)$-branes), $(N_1-\bar{N}_1)$ Dp-branes (or $(\bar{N}_1-N_1)$ $\bar{\mathrm{D}}p$-branes) or an empty space-time respectively. The tachyon vev decouples in all these three cases at the bottom of the tachyon potential as expected.

To summarize, we have interpreted the supergravity solution given in (1) with the metric having the isometry ISO$(p-4,1)\times$SO$(4)\times$SO$(9-p)$ as the intersecting Dp-$\bar{\mathrm{D}}p$ and D$(p-4)$-$\bar{\mathrm{D}}(p-4)$-brane system. The five parameters appeared in the supergravity solution were then naturally interpreted as related to the five microscopic physical parameters of the system namely, the number (N_1) of Dp-branes, the number $(\bar{N}_1)$ of $\bar{\mathrm{D}}p$-branes, the number (N_2) of D$(p-4)$-branes, the number $(\bar{N}_2)$ of $\bar{\mathrm{D}}(p-4)$-branes and the tachyon vev T. Based on the physical properties of the solution and the characteristic behavior of tachyon condensation, we have related the supergravity parameters with the microscopic physical parameters of the system. We have obtained the ADM mass representing the total energy per unit p-brane volume of the system from the supergravity configuration and related it with the microscopic physical parameters using the previously mentioned relations. We have shown that the ADM mass as well as the solution produce all the required properties of the tachyon condensation showing that the proposed relations capture the right picture of the tachyon condensation of the intersecting brane-antibrane system in the supergravity or closed string description.

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